

# Curve fitting for teaching labs

curve.fit fits data with uncertainties in  $x$ ,  $y$  or both, by least squares or orthogonal distance regression. Its numerical method is validated against published reference results, and each fit exports as a print-ready PDF report. It runs in a web browser.

**Free to use, with no logins, seat licenses or advertising.** More than one million fits since 2019; used in undergraduate labs in the US and Canada. Fitting stays free; planned paid options are administrative tools, an API and advanced features.

## COMPARED WITH COMMON ALTERNATIVES

### Spreadsheets

Trendlines and LINEST cannot use uncertainties in  $x$ , and their standard errors ignore those in  $y$ . Page 3 shows the effect.

### Python

Worth teaching where time allows. curve.fit runs ODRPACK95, the successor to the solver in `scipy.odr`, with no installation, and provides a matching script.

### Commercial software

Licenses and installations can limit where and when students work. curve.fit needs only a browser.

## VALIDATION OF THE NUMERICAL METHOD

Fits use ODRPACK95 [7, 8] through the odrpack package [9]. Each fitting mode is tested against an independent reference.

| Uncertainties             | Method                                         | Reference                                                          |
|---------------------------|------------------------------------------------|--------------------------------------------------------------------|
| none                      | Ordinary least squares                         | NIST certified values [10]; closed-form solution                   |
| $\delta y$                | Weighted least squares, weights $1/\delta y^2$ | Independent weighted regression, including parameter uncertainties |
| $\delta x$                | Orthogonal distance regression, $y$ exact      | Independent regression of $x$ on $y$ , transformed back            |
| $\delta x$ and $\delta y$ | Weighted orthogonal distance regression        | Pearson–York benchmark [2, 6]; independent York solution [2, 5]    |

NIST suite: 21 datasets (4 linear, 17 nonlinear) from 55 starting points, all within a relative tolerance of  $1 \times 10^{-6}$  of the certified parameters, standard errors and residual sums of squares [10, 11]. This shows agreement on the problems tested; it is not a NIST certification.

## USING CURVE.FIT

01

### Enter data

Type, paste, or import a delimited text file.

02

### Choose a model

35 built-in models, or a custom equation (page 2).

03

### Set uncertainties

Column, constant or percentage Rule; Absolute or Nominal.

04

### Fit and export

Parameters at  $1\sigma$  or 95%, Fit Statistics, residuals; PDF (page 4), XLSX or CSV.

Each session has its own URL and each PDF links back to its fit, so a TA can reopen any submitted result.

## TEACHING FEATURES

- Reduced  $\chi^2$  turns red as it departs from 1, prompting students to question the model or the uncertainties.
- Uncertainties are declared **Absolute** (calibrated, used as given) or **Nominal** (relative weights; parameter uncertainties scaled by  $\sqrt{\text{reduced } \chi^2}$ ), a choice students can be asked to defend.
- Fit Statistics give the  $\chi^2$  p-value, AIC and BIC; Previous ranks fits of the same data by AIC.
- Evaluate gives  $y \pm \delta y$  at any  $x$ , or  $x$  from a measured  $y$ , such as an unknown on a calibration line.
- “None” means no uncertainty was supplied, not exact data; a parameter the data cannot determine shows n/a, not zero.

Try it with data from one of your labs.

The worked example on page 3 loads at [curve.fit/pearson](https://curve.fit/pearson).  
More for instructors: [curve.fit/labs](https://curve.fit/labs).

curve.fit

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## Built-in models and custom equations

curve.fit includes 35 built-in models, among them a weighted mean, and a custom-equation option. Built-in models use analytic derivatives and choose their own starting values when the Guess field is left blank.

### BUILT-IN MODELS BY TYPICAL EXPERIMENT

| Area                                             | Models                                                                                                                                            | Typical experiments                                                                                                         |
|--------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------|
| <b>Mechanics and oscillations</b>                | Linear, Quadratic, Power Law, Oscillator, Damped Oscillator, Driven Oscillator Amplitude, Stokes Law                                              | Hooke's law, free fall, pendulum period against length, damped and driven harmonic motion, falling-ball viscometry          |
| <b>Optics and waves</b>                          | Malus's Law, Single Slit Diffraction, Two Slit Interference, Cauchy, Shifted Reciprocal, Gaussian, Gaussian on Linear Background, Lorentzian Peak | Polarizer transmission, single- and double-slit intensity, prism dispersion, thin-lens image distance, spectral line shapes |
| <b>Circuits and thermal</b>                      | Exponential Decay (Time Constant), Exponential with Offset, RLC Resonance, Natural Log, Arrhenius                                                 | RC charging and discharging, RLC resonance, Newton's law of cooling, temperature dependence of rates and conductance        |
| <b>Decay, counting and repeated measurements</b> | Exponential Decay (Half-life), Double Exponential, Poisson, Gaussian, Weighted Mean                                                               | Half-lives and two-component decays, counting statistics, combining repeated measurements weighted by $1/\delta y^2$        |
| <b>Chemistry and biology</b>                     | Saturation (Michaelis–Menten), Logistic 4PL (Hill), Logistic 5PL, Logistic, Logistic with Baseline, Arrhenius                                     | Enzyme kinetics, binding and dose–response, growth and melting curves, rate constants against temperature                   |
| <b>General</b>                                   | Cubic, Quartic, Inverse, Power Law with Offsets, Power Law with Vertical Offset, X Log X                                                          | Calibration curves and empirical relationships                                                                              |

Each model's equation is shown in the interface and printed on the report (page 4). Each also has a page at [curve.fit/models](https://curve.fit/models) with parameter meanings and fitting tips.

### CUSTOM EQUATIONS

- **Fit the form the theory gives.** Students type the equation from the lab manual instead of linearizing it, which would distort the uncertainties.
- **Syntax.** Python syntax in  $x$  and up to five parameters, A–E;  $\wedge$  and  $**$  both work. Functions: sin, cos, tan, their inverse, hyperbolic and inverse hyperbolic forms, coth, exp, log or ln, log10, sqrt, abs, sinc, erf, erfc, floor, ceil, fact, degrees, radians, besseli0, besseli1; constants pi and  $\pi$ .
- **Checked as you type.** Syntax errors are flagged while editing, with a suggested correction or a matching built-in model.
- **Starting values, fixed parameters and limits.** Enter guesses for custom equations; fix any parameter at a theoretical value or keep it within limits, or fix all of them to plot the prediction against the data with residuals and goodness of fit.
- **Validated.** The 21-dataset NIST suite (page 1) runs through the custom-equation path, with numerical derivatives. Saved fits and reports show the equation typeset.

#### EXAMPLES

$$A \cdot \exp(-x/B) \cdot \cos(2\pi x/C + D) + E$$

Damped oscillation with an offset, fitting the period  $C$  directly

$$A/\sqrt{1 + (x/B)**2}$$

RC low-pass filter gain against frequency;  $B$  is the corner frequency

$$A \cdot x \cdot x \cdot B$$

The same as the built-in Power Law, typed by hand. Useful when students should write the model themselves

**Need a model that isn't listed?** Suggestions for new built-in models and functions are welcome.

[info@curve.fit](mailto:info@curve.fit)

# Pearson’s data with York’s weights

Pearson used these points to illustrate fitting by perpendicular distance [1]; York added weights for both coordinates [2]. The set is a standard benchmark for fits with uncertainties in both variables [3–6] and loads at [curve.fit/pearson](#).

| i  | x     |      |        | y     |     |        |
|----|-------|------|--------|-------|-----|--------|
|    | value | w    | σ      | value | w   | σ      |
| 1  | 0.0   | 1000 | 0.0316 | 5.9   | 1   | 1.00   |
| 2  | 0.9   | 1000 | 0.0316 | 5.4   | 1.8 | 0.745  |
| 3  | 1.8   | 500  | 0.0447 | 4.4   | 4   | 0.500  |
| 4  | 2.6   | 800  | 0.0354 | 4.6   | 8   | 0.354  |
| 5  | 3.3   | 200  | 0.0707 | 3.5   | 20  | 0.224  |
| 6  | 4.4   | 80   | 0.112  | 3.7   | 20  | 0.224  |
| 7  | 5.2   | 60   | 0.129  | 2.8   | 70  | 0.120  |
| 8  | 6.1   | 20   | 0.224  | 2.8   | 70  | 0.120  |
| 9  | 6.5   | 1.8  | 0.745  | 2.4   | 100 | 0.100  |
| 10 | 7.4   | 1    | 1.00   | 1.5   | 500 | 0.0447 |

**Table 1.** Pearson’s data with York’s weights, as tabulated in [6].  $\sigma = 1/\sqrt{w}$  gives the  $\delta x$  and  $\delta y$  entered in curve.fit.

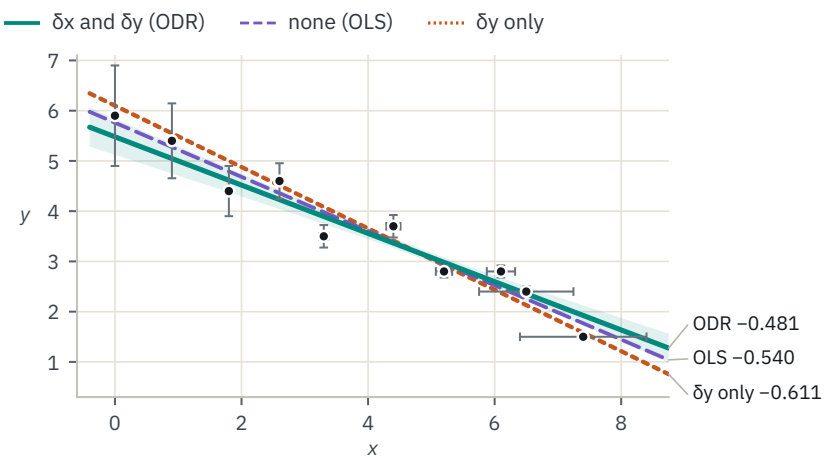


**Figure 1.** Fits to the same data with three uncertainty settings. Error bars  $\pm 1\sigma$ ; shading, the  $\pm 1\sigma$  model band of the ODR line, as curve.fit plots it.

## COMPARISON OF UNCERTAINTY SETTINGS

| curve.fit setting                   | Method                          | Slope                                   | Intercept                            | Red. $\chi^2$ | Slope vs ref. |
|-------------------------------------|---------------------------------|-----------------------------------------|--------------------------------------|---------------|---------------|
| $\delta x$ None · $\delta y$ None   | Least squares in y              | $-0.53958 \pm 0.0421$                   | $5.7612 \pm 0.189$                   | —             | +12.3%        |
| $\delta x$ None · $\delta y$ Table  | Weighted least squares in y     | $-0.61081 \pm 0.0623$                   | $6.1001 \pm 0.424$                   | 4.29          | +27.1%        |
| $\delta x$ Table · $\delta y$ Table | <b>Weighted ODR</b>             | <b><math>-0.48053 \pm 0.0706</math></b> | <b><math>5.4799 \pm 0.359</math></b> | 1.48          | <b>0.0%</b>   |
| Published reference                 | York [2]; Cantrell [6], Table 2 | $-0.48053 \pm 0.0706$                   | $5.4799 \pm 0.359$                   | —             | —             |

Computed by curve.fit (ODRPACK95 [7, 9]); all values agree with Cantrell [6], Table 2. Slope vs ref.: relative difference in slope magnitude. Uncertainties are  $1\sigma$  in curve.fit’s **Nominal** mode (the default, and the convention of [6]):  $\delta x$  and  $\delta y$  set relative weights, and parameter uncertainties are scaled by  $\sqrt{(\text{reduced } \chi^2)}$ . In **Absolute** mode,  $\delta x$  and  $\delta y$  are taken as calibrated standard uncertainties and not rescaled: weighted ODR slope  $\pm 0.0580$ , intercept  $\pm 0.295$ ;  $\delta y$  only  $\pm 0.0301$ ,  $\pm 0.205$ . The mode changes the reported uncertainties, not the fit. With no uncertainties supplied, they come from the residual scatter.

## OBSERVATIONS

**Weighting by  $\delta y$  alone moves the slope further from the reference** (27% steeper, against 12% unweighted): it favors points 9 and 10, whose x values are the least certain. Reduced  $\chi^2$  rises to 4.29.

**With both uncertainties, reduced  $\chi^2 = 1.48$  on 8 degrees of freedom** ( $P \approx 0.16$ ): no strong evidence that the uncertainties are underestimated. Nominal uncertainties are  $\sqrt{1.483} = 1.218$  times the Absolute ones.

**ODR is symmetric in x and y:** exchanging the variables gives the same line. Least squares is not ( $-0.540$  for y on x,  $-0.566$  for x on y) [6].

## REPRODUCING IN CURVE.FIT

1. Open [curve.fit/pearson](#).
2. Confirm **Linear**,  $\delta x$  and  $\delta y$  set to **Table**, and **Nominal**.
3. Fit, and compare with the table above.
4. On Setup, choose **Absolute** or set  $\delta x$  to **None**, and fit again.

## REFERENCES

[1] K. Pearson, “On lines and planes of closest fit to systems of points in space,” *Philos. Mag.* (Ser. 6) **2**, 559 (1901). doi:10.1080/14786440109462720

[2] D. York, “Least-squares fitting of a straight line,” *Can. J. Phys.* **44**, 1079 (1966). doi:10.1139/p66-090

[3] J. H. Williamson, “Least-squares fitting of a straight line,” *Can. J. Phys.* **46**, 1845 (1968). doi:10.1139/p68-523

[4] B. C. Reed, “Linear least-squares fits with errors in both coordinates,” *Am. J. Phys.* **57**, 642 (1989). doi:10.1119/1.15963

[5] D. York, N. M. Evensen, M. López Martínez and J. De Basabe Delgado, “Unified equations for the slope, intercept, and standard errors of the best straight line,” *Am. J. Phys.* **72**, 367 (2004). doi:10.1119/1.1632486

[6] C. A. Cantrell, “Technical Note: Review of methods for linear least-squares fitting of data and application to atmospheric chemistry problems,” *Atmos. Chem. Phys.* **8**, 5477 (2008). doi:10.5194/acp-8-5477-2008

[7] J. W. Zwolak, P. T. Boggs and L. T. Watson, “Algorithm 869: ODRPACK95,” *ACM Trans. Math. Softw.* **33**, 27 (2007). doi:10.1145/1268776.1268782

[8] P. T. Boggs, R. H. Byrd, J. E. Rogers and R. B. Schnabel, *User’s Reference Guide for ODRPACK Version 2.01: Software for Weighted Orthogonal Distance Regression*, NISTIR 4834 (1992).

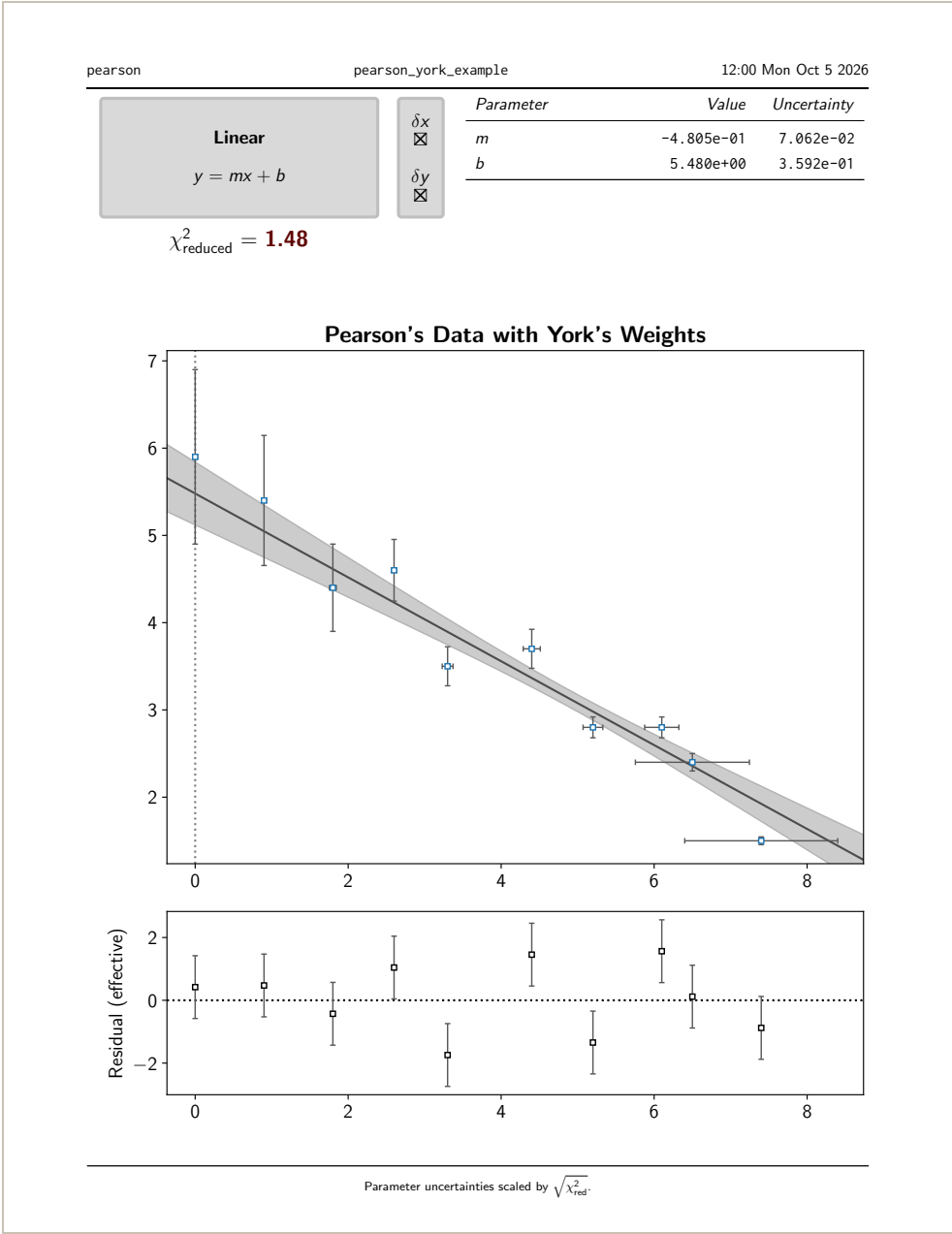
[9] H. Vale, odrpack, ODRPACK95 for Python. [pypi.org/project/odrpac](#)

[10] NIST Statistical Reference Datasets (StRD), linear and nonlinear regression, NIST Standard Reference Database 140. [itl.nist.gov/div898/strd](#)

[11] B. D. McCullough, “Assessing the reliability of statistical software: Part I,” *Am. Stat.* **52**, 358 (1998). doi:10.1080/00031305.1998.10480597

# Example report

The Single PDF export of the page 3 fit, unedited, shown at 60% scale. Reports are typeset with LaTeX, with vector graphics, on one Letter page.



Pearson's Data with York's Weights

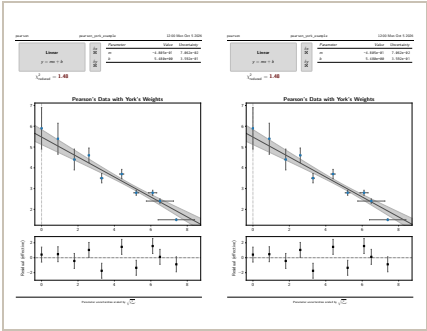
5

Effective residuals with uncertainties

6

The uncertainty mode, stated on the report: here Nominal, scaled by  $\sqrt{(\text{reduced } \chi^2)}$

- 1 Session and fit IDs, linked back to the fit; timestamp
- 2 Model, uncertainty settings, and parameters with  $1\sigma$  uncertainties (95% when selected)
- 3 Reduced  $\chi^2$  for the weighted fit
- 4 Data with  $\pm 1\sigma$  error bars, the fit and its  $\pm 1\sigma$  model band. Title and axis labels are set by the student; LaTeX, Greek letters and symbols such as  $^\circ$  and  $\mu$  are supported
- 5 Effective residuals with uncertainties
- 6 The uncertainty mode, stated on the report: here Nominal, scaled by  $\sqrt{(\text{reduced } \chi^2)}$



## DOUBLE PDF

The same report twice on one landscape sheet, so lab partners each get a copy from one print job.

Each fit also offers an XLSX workbook with full-precision tables and the plot, a Browser Report, and the data as CSV.

Open this example: [curve.fit/pearson](https://curve.fit/pearson)